EE215 – FUNDAMENTALS OF ELECTRICAL ENGINEERING

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WEEK 8
FIRST ORDER CIRCUIT RESPONSE

May 21st , 2010

QUESTIONS TO ANSWER

- First order circuits
  - What is the definition of first order circuit?
  - What are the difference between natural and step response?
  - How to analyze and solve the natural response of RL and RC circuits?
  - How to analyze and solve the step response of RL and RC circuits?

FIRST ORDER CIRCUITS RESPONSE

May 21^{st} , 2010

NATURAL AND STEP RESPONSE OF FIRST-ORDER CIRCUITS

- Def. – First-Order Circuit:

- Examples:
  - RL circuits: only sources, resistors (R), inductors (L)
  - RC circuits: only sources, resistors (R), capacitors (C)
  - RLC?

NATURAL AND STEP RESPONSE OF FIRST-ORDER CIRCUITS

- Def. – Response:

- Def. – Natural Response:

- Def. – Step Response:

- Also distinguish:
  - Transient: currents and voltages are changing.
  - Steady-state: currents and voltages have reached DC values.

NATURAL RESPONSE OF RL CIRCUIT

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- We are interested in the natural response of the RL circuit for $t \geq 0$.
  - –

NATURAL RESPONSE OF RL CIRCUIT

- At $t = 0^{-}$ and $t = 0^{+}$ circuit can be simplified:

- We want to find $i(t)$ and $v(t)$. KVL:.

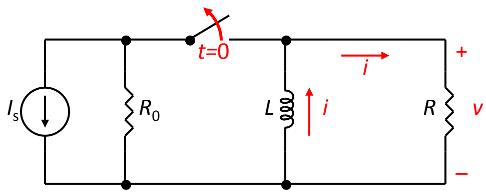
NATURAL RESPONSE OF RL CIRCUIT

- Solve $L\, di/dt + Ri = 0$ for $i(t)$:
  - Rearranging…
  - Integral…
  - Integrating…
  - Solving for $i(t)$

NATURAL RESPONSE OF RL CIRCUIT

- What is $i(0)$?
- We know
- We also know
- So, the natural response of the RL circuit is:

NATURAL RESPONSE OF RL CIRCUIT



- Example: I_s=1A, R=10Ω , L=1mH

NATURAL RESPONSE OF RL CIRCUIT

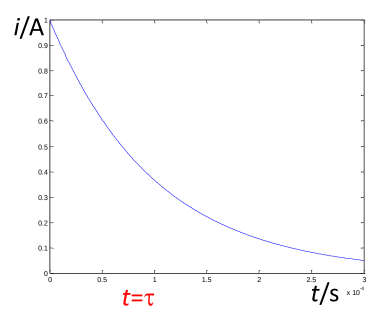
- Power dissipated in R:
- Energy dissipated in R:
- Energy delivered by L:

TIME CONSTANTS

- $i(t) = I_0\, e^{-R/L\, t}$ is the natural response of RL circuit.
- R/L
  - –
  - –
- Def. – <u>Time Constant</u>
- Thus,.

TIME CONSTANTS

- Consider the tangent of natural response at $t = 0$:



- The tangent is given by
  - This provides a simple experimental method to determine τ.

NATURAL RESPONSE OF RC CIRCUIT

- Note: the analysis is very similar to the RL circuit.

NATURAL RESPONSE OF RC CIRCUIT

- At $t = 0^-$ and $t = 0^+$ circuit can be simplified:

- We want to find $v(t)$ and $i(t)$..

NATURAL RESPONSE OF RC CIRCUIT

- In analogy to analysis of RL circuit, we obtain:

- Energy dissipated by $\boldsymbol{R}$:

- Energy delivered in $\boldsymbol{C}$:

NATURAL AND STEP RESPONSE OF FIRST-ORDER CIRCUITS

- Circuits with one capacitor or one inductor are called *first order circuits*, because they give rise to first order linear differential equations.
- Engineers have a love/hate relationship with differential equations. They describe the things we do engineering with, like circuits, so we need to understand them and obtain solutions. But we are not math majors! All we want is a solution! We don't care that there are eight different ways to solve a differential equation, we just want one way that works!
- So what we'll do with first order circuits is first write the circuit equations from the circuit. Then we'll solve them formally, just once to prove we can do it. Then we'll develop a short cut so that all we have to do is write the *form of the solution* and fill in some numbers from looking at the circuit. And that's the method you really use to solve first order circuits.

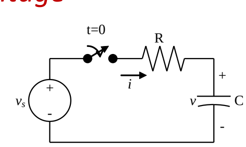
STEP RESPONSE FOR RC CIRCUIT (1)

- Let's start by considering the circuit before t = 0. There is clearly no current, but what about capacitor voltage? Well, it could be anything, really, in this case, so the problem has to specify this *initial condition*. Let $v(0^-) = 0$. $t = 0^-$ means the time just before t = 0.

STEP RESPONSE FOR RC CIRCUIT (2)

- Now consider the circuit an instant after the switch closes. (This would be $t = 0^+$.) Recall that

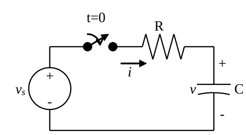
No time goes by from 0^- to 0^+, so any change in voltage across the capacitor would require infinite current, which is not possible. Therefore**,** *capacitors cannot change voltage instantaneously.*



STEP RESPONSE FOR RC CIRCUIT (3)

- That means all the source voltage appears across the resistor, by KVL. Then the current is

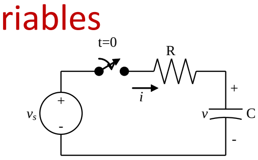
So the current did change instantaneously through the capacitor. We could calculate the rate of change of capacitor voltage as



STEP RESPONSE FOR RC CIRCUIT (4)

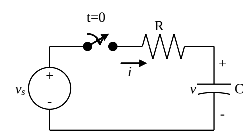
- but let's try for the circuit equation instead. KCL at the capacitor is

Let's solve for the derivative, separate variables and integrate



STEP RESPONSE FOR RC CIRCUIT (5)

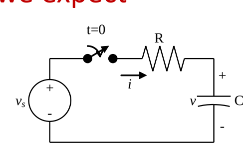
- Let's solve for the derivative, separate variables and integrate



STEP RESPONSE FOR RC CIRCUIT (6)

- We can find the value of *B* from considering the initial condition,

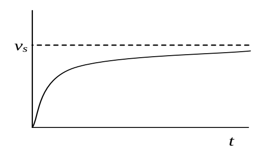
We can check this by considering what happens as *t* goes to infinity. In steady state, with a constant source, we expect the capacitor current to go to zero.



STEP RESPONSE FOR RC CIRCUIT (7)

- Then from the circuit equation,

Note that *at t = infinity the capacitor looks like an open circuit*. Replacing the capacitor with an open circuit is an easy way to obtain steady state values. Anyway, the complete solution is

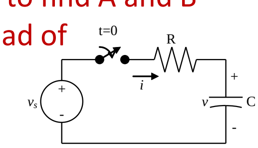


GENERAL SOLUTION FOR RC CIRCUIT

- ANY circuit with one capacitor and a resistor has a solution of the form

Let's rewrite this slightly:

Of course *A* and *B* can be found from v_{oc} and $v(0^+)$, but you don't really need to remember that. It's easier to find A and B from $v(0^+)$ and $v(\infty)$, doing a little algebra instead of memorizing an equation.



GENERAL SOLUTION FOR RC CIRCUIT (COOKBOOK)

1.

2.

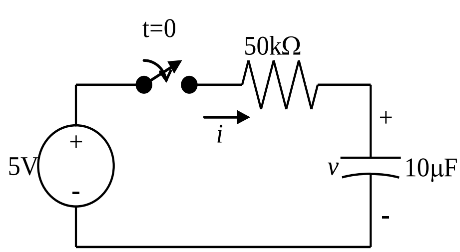
3.
4.

5.

6.

GENERAL SOLUTION FOR RC CIRCUIT (COOKBOOK)

- **Cookbook Example:**



Given $v(0^-) = 0$ V, find $i(t)$ for $0 < t < \infty$.

GENERAL SOLUTION FOR RC CIRCUIT (COOKBOOK)

1.
2.
3.
4.
5.

Solution

6. (Don't forget this step!)